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#] Probability Theory.

*) **Tossing of N Coins:** Say, there are N identical coins which are tossed simultaneously number of times. Say we have r heads uppermost, which also means $(N-r)$ tails uppermost.

The number of ways in which r heads (and $N-r$ tails) can occur in N identical coins is

$$ {}^NC_r = \frac{N!}{r!\,(N-r)!} $$

The total number of ways of all possible combinations is

$$ \sum_{r=0}^{N} {}^NC_r = (1+1)^N = 2^N. $$

∴ The Probability of having r heads (and $N-r$ tails) is

$$ P(r) = \frac{{}^NC_r}{\sum_{r=0}^{N} {}^NC_r} = \frac{N!}{r!\,(N-r)!}\cdot\frac{1}{2^N} \quad \text{---(i)} $$

#] **Most Probable Combination:** The combination of heads and tails which is most likely to occur is the one for which $P(r)$ is maximum. Now, $P(r)$ is maximum when ${}^NC_r$ is maximum. Since ${}^NC_r$ is maximum when $r = N/2$. Thus, the most probable combination is the one having as many heads as the tails.

Putting $r = N/2$ in eqn. (i); we get

$$ P_{max} = \frac{N!}{(N/2)!\,(N/2)!}\cdot\frac{1}{2^N}. $$

#] **Least Probable Combination:** The combination of heads and tails least likely to occur is the one having minimum $P(r)$. Again the probability $P(r)$ is minimum when ${}^NC_r$ is minimum, that is, when $r=0$ or N. Thus, the least probable combination is the one having all heads or all tails.

Putting $r=0$ or $r=N$ in eqn (i); we get

$$ P_{min} = \frac{N!}{0!\,N!}\,\frac{1}{2^N} = \frac{1}{2^N} \quad [\because 0! = 1]. $$

Teacher's Signature

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